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LETTER TO THE EDITOR

Relationship between coherent states and intelligent states as applied to spins

L Kolodziejczyk†

Theoretical Physics Department. University of Lodz, Lodz, Poland

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Abstract. This letter shows that coherent states as defined by Mikhailov, may be considered in certain special cases as intelligent states.

According to Aragone *et al* (1974) the intelligent states are those which verify the Heisenberg equality for the spin operators (J_x, J_y, J_z)

$$(\Delta J_x)^2_w(\Delta J_y)^2_w = \tfrac{1}{4}\langle w|J_2|w\rangle^2. \tag{1}$$

The problem of intelligence for N two-level spins has, recently, been discussed by Vetri (1975). This paper deals with the more general case, namely N $(2s+1)$-level spins. To do this, we apply coherent states proposed by Mikhailov (1973).

Here in brief is the Mikhailov theory.

If we take a_μ, $a^\dagger_\mu$ couples of shift operators ($\mu = -s, \ldots s$), s integer or half-integer

$$[a_\mu, a^\dagger_{\mu'}] = \delta_{\mu\mu'} \tag{2}$$

we can construct operators:

$$J_3 = \sum_{\mu\mu'} a^\dagger_\mu\langle s\mu|\hat{J}_3|s\mu'\rangle a_{\mu'} = \sum_\mu \mu a^\dagger_\mu a_\mu \tag{3a}$$

$$J_+ = \sum_{\mu\mu'} a^\dagger_\mu\langle s\mu|\hat{J}_+|s\mu'\rangle a_{\mu'} = \sum_\mu [(s-\mu)(s+\mu+1)]^{1/2} a^\dagger_{\mu+1}a_\mu \tag{3b}$$

$$J_- = (J_+)^\dagger \tag{3c}$$

$$[J_3, J_\pm] = \pm J_\pm, \qquad [J_+, J_-] = 2J_3. \tag{3d}$$

The last equalities indicate that operators (J_1, J_2, J_3) may be regarded as the rotation operators.

Mikhailov coherent states attached to the rotation are:

$$|\alpha s\rangle = \exp(-\tfrac{1}{2}n^{2s})\prod_\mu \exp(\alpha_{s\mu}a^\dagger_\mu)|0\rangle \tag{4}$$

where

$$\alpha_{s\mu} = (\alpha_+)^{s+\mu}(\alpha_-)^{s-\mu}\left[\begin{pmatrix} 2s \\ s-\mu \end{pmatrix}\right]^{1/2}$$

† Present mailing address: 12128 Ivy Place, Los Angeles, California 90064, USA.

(α_+, α_-) spin components

$$n^{2s} = (|\alpha_+|^2 + |\alpha_-|^2)^{2s} = \sum_\mu |\alpha_{s\mu}|^2.$$

One can easily check that

$$a_\mu|\alpha s\rangle = \alpha_{s\mu}|\alpha s\rangle. \tag{5}$$

We see that Mikhailov space is Glauber-like.

Using his model, Mikhailov obtained the following formulae for certain average values:

$$\langle\alpha s|J_i|\alpha s\rangle = \langle J_i\rangle = 2sn^{2s-1}j_i \tag{6a}$$

$$\sum_{i=1}^{3} \langle J_i\rangle^2 = s^2 n^{4s} \tag{6b}$$

$$(\Delta J_i)^2 = sn^{2s}\left[2(2s-1)\left(\frac{j_i}{n}\right)^2 + \frac{1}{2}\right] \tag{6c}$$

$$j_1 = \mathrm{Re}(\alpha_+\alpha_-), \qquad j_2 = \mathrm{Im}(\alpha_+\alpha_-)$$

$$j_3 = \tfrac{1}{2}(|\alpha_+|^2 - |\alpha_-|^2). \tag{6d}$$

From (1) and (6) we can easily construct the equation

$$4(2s-1)^2 \frac{\langle J_1\rangle^2}{4s^2n^{4s}} \frac{\langle J_2\rangle^2}{4s^2n^{4s}} + \frac{s}{2} = 2s\frac{\langle J_3\rangle^2}{4s^2n^{4s}}. \tag{7}$$

From (6*b*) we can write

$$\frac{\langle J_1\rangle^2}{\Sigma\langle J_i\rangle^2} = \frac{\langle J_1\rangle^2}{s^2n^{4s}} = \sin^2\theta\cos^2\bar{\phi} \tag{8a}$$

$$\frac{\langle J_2\rangle^2}{\Sigma\langle J_i\rangle^2} = \frac{\langle J_2\rangle^2}{s^2n^{4s}} = \sin^2\theta\sin^2\bar{\phi} \tag{8b}$$

$$\frac{\langle J_3\rangle^2}{\Sigma\langle J_i\rangle^2} = \frac{\langle J_3\rangle^2}{s^2n^{4s}} = \cos^2\theta \tag{8c}$$

$(\theta, \bar{\phi})$ is the direction along which the expected value of J reaches its maximum.

If we substitute (8) in (7) we have

$$\tfrac{1}{4}(2s-1)^2\sin^4\theta\cos^2\bar{\phi}\sin^2\bar{\phi} + \tfrac{1}{2}s\sin^2\theta = 0. \tag{9}$$

This equation has only one root $\sin\theta = 0$ ($\theta = 0$ or π). This means that intelligent states exist when the z axis is oriented in the direction of the greatest value of J.

In conclusion, we have shown that the coherent states defined by Mikhailov (1973) can be used to explain the relationship between coherent and intelligent states in terms of spins.

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